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SVPWM control system of IDW synchronous motor based on suboptimal solution set

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Abstract. affected by friction, speed of linear motor in the low speed motion is often not stable, so it is not easy to ensure steady precision. In this paper, a kind of pole assignment method of self-adaptive state space which is able to ensure steady state precision is proposed to improve steady control precision of DC linear motor, of which basic idea is to adjust and increase gain matrix with self-adaption of feedback gain matrix realize the online estimation for feedback matrix and gain matrix with self-adpative regulator, compensate for steady state error and obatin pole assignment algorithm of self-adaptive state space to overcome the influence of friction on motor performance. Results of Matlab simulation experiment of theoretical analysis indicate that the online esimation and compensation of steady state precision can be realized with this control algorithm which can follow the speed of motor quickly and accurately Its system has better dynamic and steady state performances and the closed-loop system has stronger robustness.

Key words. DC linear motor, steady state error compensation, self-adpative regulator, pole assignment control of state space.

1. Introduction

DC Linear Motor (DCLM) is a special motor which converts electrical energy into mechanical energy of linear motion directly without need of any intermediate conversion mechanism and has many advantages [1–4], such as simple structure, low noise, fast speed, low cost, reliable operation, high efficiency and energy saving and flexible assembly, which is widely used [5, 6] in automatic production, civil aviation transportation, military and other fields. However, from the perspective of linear motor with contact motion, friction is an important factor affecting the control performance. DCLM can not output steady speed in low speed motion and even stick and slip [7, 8] for the influence of friction. In addition, since friction is not

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a continuous process at zero point of motor speed, larger tracking error will occur in reversing of motor in forward and reverse motions so that tracking precision of system is reduced. Thus, steady state precision needs to be compensated to improve control performance of DCLM.

Current researches are based on control of DCLM with the control algorithms and all the algorithms have a common characteristic, namely they are intelligent control algorithms almost. Those algorithms include complex calculations and can not meet requirements of real-time control of motor. Moreover, all of them have not analyzed the internal structure of DCLM but only controlled motor based on the "black box principle" which is known as "classical theory" in the control field. Thus, the pole assignment algorithm of self-adaptive state space proposed in this paper is determined by the position of system pole, in which poles of the closed-loop system are assigned to the planned positions. In fact, it is equal to promoting the comprehensive dynamic performance of system to reach expectation and making use of controller of self-adaptive pole assignment state to deduce the control law by which the optimal feedback gain matrix of system can be obtained so that system performance can be improved. By means of reasonable selection of gain matrix of system with the controller of pole assignment state, tracking performance and steady state error of system can be improved, zero-pole position of closed-loop system can be arranged properly and requirements of dynamic performance of system can be met. It is shown from theory derivation and computer simulation results that this control algorithm has better control quality.

2. Mathematical modeling of DCLM

2.1. Operating principle of DCLM

In this paper, the permanent magnetic DCLM is selected. As shown [1] in Fig. 1, magnetic pole of the motor is made of permanent magnet and two permanent magnets with synclastic polarity are equipped on the both ends of soft iron stand. When DC current is input into moving coil, electromagnetic force will be produced. As long as the electromagnetic force is greater than the static friction resistance of sliding rail, the coil will move along the rail and its motion direction is determined by the left-hand rule. Magnitude and direction of the electromagnetic force can be changed by changing the magnitude and direction of the DC current in the coil:

$$F = B_\delta l W I_a \,. \tag{1}$$

In the formula, B_δ is the magnetic flux density in the space of the coil; l is the effective length per average number of turns of coil conductor in the magnetic field; W is the number of turns of coil; I_a is the current in the coil.

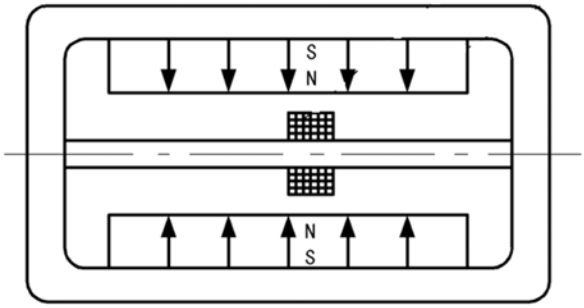

Fig. 1. Structure Chart of Permanent Magnetic DCLM

2.2. Mathematical model of DCLM

In the actual operating process, motor will be influenced by external forces, such as sliding friction, viscous friction. Thus, these external forces are expressed as $N_{(S)}$ uniformly and assume that the magnitude is directly proportional to speed and the proportional coefficient is C. Then block diagram of simplified mathematical model of DCLM is shown in Fig.2.

In the diagram, K_E is the back EMF coefficient related to speed, namely, $K_E = k_b B_\delta l N$. τ_L is the electromagnetic time constant of the coil loop and the value can be calculated as $\tau_L = \frac{L}{R}$. Transfer function of DCLM can be obtained by means of Fig. 2.

$$G_{(s)} = \frac{y_{(s)}}{U_{(s)}} = \frac{k_m}{(ms^2 + cs + k)(1 + \tau_L s)R + k_m k_E s} . \tag{2}$$

When the coil inductance L is neglected in the low frequency stage, the transfer function can be simplified as:

$$G_{(s)} = \frac{\frac{k_m}{Rm}}{s^2 + \frac{Rc + k_m k_E}{Rm} s + \frac{k}{m}} . \tag{3}$$

In the formula, k_m and k_E are the back EMF coefficients related to acceleration and speed respectively, R is resistance coefficient of coil loop, m is total mass of active cell of the linear motor, c is the resistance coefficient and k is the elastic coefficient of spring.

3. Design for pole assignment controller of self-adaptive state space

3.1. State space model of DCLM

In order to combine with the actual needs of motor, open-loop expression of mathematical model of motor can be obtained after setting internal parameters of

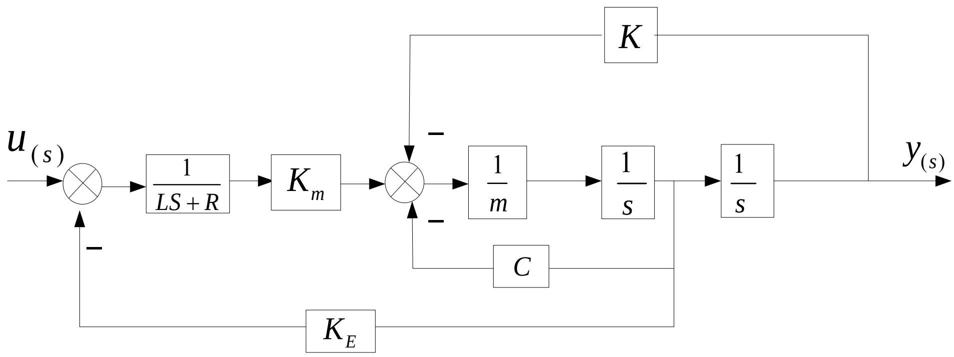

Fig. 2. Block Diagram of Simplified Mathematical Model of DCLM

motor;

$$G_{0(S)} = \frac{k_1}{s(s+2)} \,. \tag{4}$$

In the formula, $\frac{k_1}{2}$ is the open-loop amplification coefficient of DCLM system, and state space expression of DCLM can be obtained with open-loop expression in formula (4) after selecting speed Ω, acceleration a and input signal n of DCLM as the variation of state space:

$$\begin{cases} \dot{\vec{\mathbf{x}}} = \begin{bmatrix} \dot{\Omega} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -k_1 & 0 \end{bmatrix} \begin{bmatrix} \Omega \\ \alpha \end{bmatrix} + \begin{bmatrix} 0 \\ k_1 \end{bmatrix} [n] \,, \\ \dot{\vec{\mathbf{y}}} = [\dot{y}] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \Omega \\ \alpha \end{bmatrix} . \end{cases} \tag{5}$$

Above equations are written as following vector form to facilitate the subsequent derivation:

$$\begin{cases} \dot{\vec{\mathbf{x}}} = \vec{\mathbf{A}}\vec{\mathbf{x}} + \vec{\mathbf{B}}\vec{\mathbf{u}} \,, \\ \dot{\vec{\mathbf{y}}} = \vec{\mathbf{C}}\vec{\mathbf{x}} \,. \end{cases} \tag{6}$$

Where $\vec{\mathbf{x}}$ is two-dimensional state, namely the speed and acceleration states of motor, $\vec{\mathbf{u}}$ is the input, namely the input signal state of motor and $\dot{\vec{\mathbf{y}}}$ is output of motor, namely the rotating speed of motor, with $\vec{\mathbf{A}}$ as 2×2 system matrix, $\vec{\mathbf{B}}$ as 2×1 input matrix, $\vec{\mathbf{C}}$ as 1×2 output matrix and coupled matrix $\vec{\mathbf{D}} = 0$.

3.2. Pole assignment of self-adaptive state space

Performance of the control system (such as overshoot and time in transient process) mainly depends on the position of system pole. After assigning the poles of closed-loop system in the planned position and as selecting a set of expectation poles, namely the characteristic values, as performance indexes, the desired dynamic performance can be obtained by integrating the control of a state feedback of DCLM system and selecting feedback gain matrix. According to the steady state error output by DCLM, the control law can be deduced on line in real time to correct feedback gain matrix, obtain the expected position of closed-loop pole and acquire the satisfactory control effect. At the same time, the appropriate gain matrix is selected according to calculation to ensure obtaining the better dynamic and static characteristics of control system.

Theorem [15] of pole assignment of DCLM: for m-dimensional linear time invariant controlled system with single input in continuous time:

$$\dot{\vec{\mathbf{x}}} = \vec{\mathbf{A}}\vec{\mathbf{x}} + \vec{\mathbf{B}}\vec{\mathbf{u}}\,. \tag{7}$$

The necessary and sufficient condition for arbitrary assignment of m poles, namely the characteristic values of system is that $(\vec{\mathbf{A}}, \vec{\mathbf{B}})$ can be controlled entirely.

According to state space model of DCLM, pole assignment algorithm of self-adaptive state space is shown as follows:

Step 1 Calculate the controllability of $(\vec{\mathbf{A}}, \vec{\mathbf{B}})$. If controllable entirely, then go to the next step, otherwise, turn to step 10.

If $(\vec{\mathbf{A}}, \vec{\mathbf{B}})$ can be controlled entirely, then there is nonsingular transformation: $\mathbf{x} = \mathbf{P}\overline{\mathbf{x}}$. Formula (6) is converted into controllable standard type I:

$$\begin{cases} \dot{\overline{\vec{\mathbf{x}}}} = \overline{\vec{\mathbf{A}}}\,\overline{\vec{x}} + \overline{\vec{\mathbf{B}}}\vec{\mathbf{u}}\,, \\ \dot{\vec{\mathbf{y}}} = \overline{\vec{\mathbf{C}}}\,\overline{\vec{\mathbf{x}}}\,. \end{cases} \tag{8}$$

In the formula, $\overline{\vec{\mathbf{A}}} = \mathbf{P}^{-1}\vec{\mathbf{A}}\mathbf{P} = \begin{bmatrix} 0 & 1 \\ -k_1 & -2 \end{bmatrix}$ and $\overline{\vec{\mathbf{B}}} = \mathbf{P}^{-1}\vec{\mathbf{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. $\overline{\vec{\mathbf{C}}} = \vec{\mathbf{C}}\mathbf{P}^{-1} = (k_1, 0)$, then closed-loop transfer function of DCLM is:

$$G_{(s)} = \vec{\mathbf{C}}(s\mathbf{I} - \overline{\vec{\mathbf{A}}})^{-1}\overline{\vec{\mathbf{B}}} = \frac{k_1}{s^2 + 2s + k_1}\,. \tag{9}$$

Step 2 Control law of feedback gain matrix is deduced after adding feedback gain matrix.

State feedback gain matrix is added: $\mathbf{m} = (m_1, m_2)$, then control law of state feedback is:

$$\vec{\mathbf{u}} = -\mathbf{m}\vec{\mathbf{x}} + \mathbf{v}\,. \tag{10}$$

In the formula, $\overline{\mathbf{m}}$ is the feedback gain matrix of 1×2 motor and $\mathbf{v}$ is the reference input. Following expressions of state space with feedback gain matrix can be obtained after substituting formula (10) into formula (6):

$$\begin{cases} \dot{\vec{\mathbf{x}}} = (\vec{\mathbf{A}} - \vec{\mathbf{B}}\mathbf{m})\vec{\mathbf{x}} + \vec{\mathbf{B}}\mathbf{v}\,, \\ \dot{\vec{\mathbf{y}}} = \vec{\mathbf{C}}\vec{\mathbf{x}}\,, \end{cases} \tag{11}$$

In the formula, $\vec{\mathbf{A}} - \vec{\mathbf{B}}\mathbf{m} = \begin{bmatrix} -2 & 1 \\ -k_1 - k_1 m_1 & -k_1 m_2 \end{bmatrix}$.

Step 3 Calculate the closed-loop transfer function of DCLM with feedback gain matrix.

Different feedback gain matrixes $\mathbf{m}$ can be synthesized by different performance indexes designed on control site in industrial processes of DCLM and the different expected performance indexes. Chengdu DCLM transfer function is shown as follows after adding feedback gain matrix:

$$G_{m(s)} = \overrightarrow{\mathbf{C}}(s\mathbf{I} - \overrightarrow{\mathbf{A}} + \overrightarrow{\mathbf{B}}\mathbf{m})^{-1}\overrightarrow{\mathbf{B}} \quad = \frac{k_1}{s^2 + (k_1 m_2 + 2)s + k_1 + k_1 m_1 + 2k_1 m_2} \,. \quad (12)$$

Closed-loop characteristic polynomial can be obtained via formula (12), as shown as follows:

$$D_{m0(s)} = s^2 + (k_1 m_2 + 2)s + k_1 + k_1 m_1 + 2k_1 m_2 \,. \quad (13)$$

Step 4 Calculate the closed-loop characteristic polynomial $D^*_{f(s)}$ determined by expected closed-loop characteristic polynomial $\{\lambda_1^* \quad \lambda_2^*\}$.

Different performance indexes are required according to needs of control site of DCLM. DCLM determined in this paper is required to dynamic expectation index $\delta_p \le 8.5\%, t_s \le 2$ under the premise of ensuring steady state precision index $k_v = 5, e_{ss} \le 0.8$. Characteristic value of closed-loop system of expectation motor determined is $\lambda_1^* = -2 + j2.46 \quad \lambda_2^* = -2 - j2.46$. Then the expected closed-loop characteristic polynomial can be expressed as:

$$D_{f(s)} = (s - \lambda_1^*)(s - \lambda_2^*) = (s + 2 - j2.46)(s + 2 + j2.46) = s^2 + 4s + 10 \,. \quad (14)$$

Step 5 After comparing the closed-loop characteristic polynomial of added feedback gain matrix with the expected closed-loop characteristic polynomial, calculate all the coefficients of feedback gain matrix.

After comparing formula (13) with formula (14), value of feedback gain matrix of motor can be expressed as: $m_1 = \frac{6}{1+k_1}, m_2 = \frac{2}{k_1}$, then $\mathbf{m} = (\frac{6}{1+k_1}, \frac{2}{k_1})$, the feedback gain matrix of DCLM can be worked out.

Step 6 Calculate the controllable and normalized transformational matrix $\overrightarrow{\mathbf{p}}$ and solve its inverse matrix $\overrightarrow{\mathbf{Q}} = \overrightarrow{\mathbf{p}}^{-1}$. Calculate the controllable and normalized feedback gain matrix $\overline{\mathbf{m}} = \mathbf{m}\overrightarrow{\mathbf{Q}}^{-1} = \mathbf{m}\overrightarrow{\mathbf{p}}$.

Follows can be worked out by means of control law of feedback gain matrix of DCLM, namely $\overrightarrow{\mathbf{u}} = -\overline{\mathbf{m}}\overline{\overrightarrow{\mathbf{x}}} + \mathbf{v} = -\mathbf{m}\overrightarrow{\mathbf{p}}\,\overrightarrow{\mathbf{x}} + \mathbf{v}$.

$$\overrightarrow{\mathbf{p}} = [\overrightarrow{\mathbf{A}}\overrightarrow{\mathbf{B}}, \overrightarrow{\mathbf{B}}] = \begin{bmatrix} k_1 & 0 \\ 0 & k_1 \end{bmatrix}$$

$$\overline{\mathbf{m}} = \begin{bmatrix} \frac{6}{1+k_1} & \frac{2}{k_1} \end{bmatrix} \begin{bmatrix} k_1 & 0 \\ 0 & k_1 \end{bmatrix} = \begin{bmatrix} \frac{6k_1}{1+k_1} & 2 \end{bmatrix} \quad (15)$$

Step 7 Calculate gain matrix K with self-adaptive regulator.

Since mathematical model of motor is a system in type /**C**ñ which requires the systematic position error, namely the steady state error of motor output speed,

$e_{sp} = 0$. Gain matrix K of system can be obtained via the definition of system error, namely $e_{sp} = \lim_{s\to 0} sE_{(s)} = \lim_{s\to 0} s(R_{(S)} - Y_{(s)}) = \lim_{s\to 0} s(R_{(S)} - KG_{m(s)}R_{(S)}) = 0$.

Step 8 Debug the gain matrix K according to systematic steady state error calculated by self-adaptive regulator and re-determine the feedback gain matrix $\mathbf{m}$ of system.

According to mathematical model of DCLM, in engineering practice, value of k_1, the open-loop amplification coefficient of motor is taken to K^*, the equivalent compensation device which is placed the front of the complete DCLM closed-loop system to reduce the error and meet requirements for stability and the dynamic performance of the system. Return to step 1 to re-assign the poles.

Step 9 Determine the value range of equivalent compensation device K^*. Then return to step 7 to recalculate gain matrix K^* until the satisfactory control effect is obtained.

Step 10 Stop calculation.

Pole assignment algorithm of self-adaptive state feedback is shown in above process. This proposed algorithm not only needs to overcome the influence of friction force on motor performance, but also ensures stability and tracking performance of the closed-loop system. Thus, steady state precision shall be solved from the perspective of the complete closed-loop system, and the structure chart of pole assignment system of self-adaptive state feedback is shown in Fig. 3.

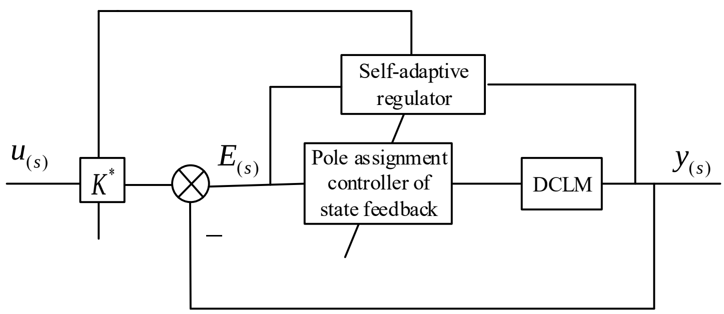

Fig. 3. Structure Chart of Pole Assignment of Self-adaptive State Feedback

Above chart shows that the complete system is composed of pole assignment controller of state feedback, self-adaptive regulator, DCLM and equivalent compensation device K^*. By means of the actual measured value of state variation of DCLM, the motor speed and acceleration are selected as the state variations in this paper, and the actual measured value of motor speed error, namely the steady state error, as a feedback is given to self-adaptive regulator which constantly adjusts pole assignment controller of state feedback and equivalent compensation device K^*, that is to say, state feedback poles are assigned until error of self-adaptive control tends to zero and satisfactory dynamic and steady state performances are obtained in motor output.

4. Simulation example

In order to show the better control performance and steady state performance of this algorithm proposed by author, DCLM are applied in two algorithms for simulation.

Parameters are selected as follows: effective stroke of DCLM: 10mm; coil inductance: 1.76mH; mass of active cell: 0.22kg; spring force coefficient: 5N/mm; number of turns of the coil: 192; magnetic induction intensity: 0.591Weber; coil resistance: 1.9Ω; average diameter of coil: $64mm$. Then simplified DCLM mathematical model is shown in formula (4) and state space model is shown in formula (5). Input of motor is required to be set as $r_{(t)} = 5 + 4t$ and dynamic expectation index is required to be $\delta_p \leq 8.5\%, t_s \leq 2$ under the premise of ensuring static index $e_{ss} \leq 0.8$.

$K^* = 10.3$, the equivalent compensation device of the DCLM control system can be obtained with online self-adaptive calculation of the algorithm proposed in this paper. Better dynamic and steady state performances of control system can be obtained when feedback gain matrix $\mathbf{m} = (5, 2)$. MATLAB/Simulnk simulation diagram of pole assignment algorithm of self-adaptive state space is shown in Fig. 4.

Step signal and speed signal set are input at the time of $k = 1$ for simulation with the different algorithms and the traditional pole assignment controller has been compared with the pole assignment controller of self-adaptive state space with ensuring steady state precision as deigned in this paper with the method of simulation, and response curve of the system is shown in figure as follows.

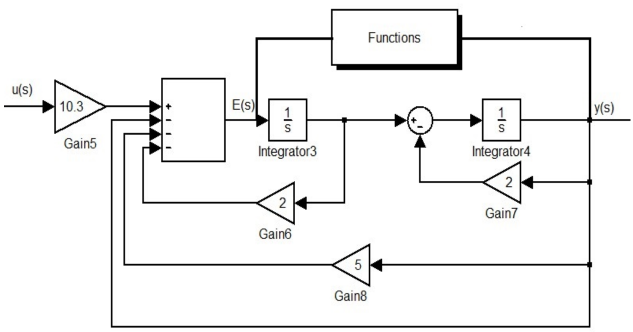

Fig. 4. MATLAB/Simulnk Simulation Diagram of Pole Assignment Algorithm of Self-adaptive State Space

It can be seen from Fig.5 that the system without any controller is not able to ensure dynamic characteristic of DCLM in step signal of motor speed set as $5r/s$; it is able to ensure dynamic performance of DCLM but can not ensure the static characteristic, with larger error in the system of traditional pole assignment algorithm of state space; however, the pole assignment algorithm of self-adaptive state space which can ensure steady state precision proposed by the author has many advantages, such as short rise time, small overshoot, short transient time, and relying on it, better dynamic performance can be obtained, and steady state

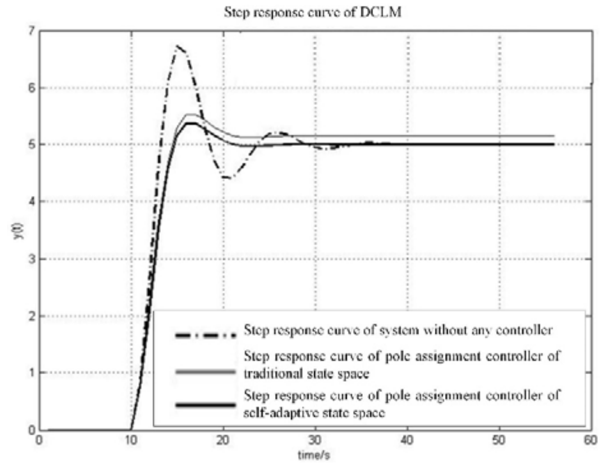

Fig. 5. Step Response Curve of DCLM

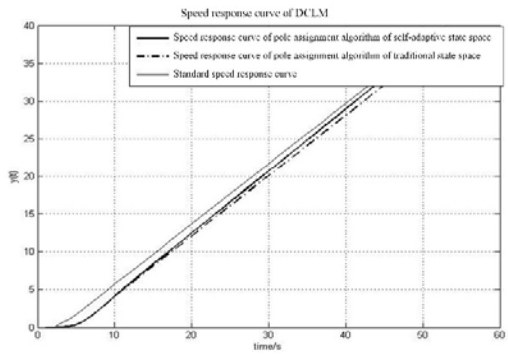

Fig. 6. Speed Response Curve of DCLM

precision of DCLM also can be ensured so that better static characteristic can be obtained. Simulation curve of DCLM speed control is shown in Fig. 6 where speed of speed ring is set as $4m/s$. It can be see from the curve that there are some errors in speed for the pole assignment controller of traditional state space of DCLM and the error is larger than that of the original system without any controller. However, the pole assignment algorithm of self-adaptive state space with ensuring steady state precision has ensured the dynamic and steady state performances of control system. With the passage of time, the speed of DCLM is infinitely close to the set speed. Thus, the pole assignment algorithm of self-adaptive state space with ensuring steady state precision has a better controller performance.

5. Conclusion

A pole assignment control algorithm of self-adaptive state space which can ensure steady state precision has been designed in this paper and it is applied in control of DCLM. Scheme of pole assignment of self-adaptive state space controlling DCLM with ensuring steady state precision is provided. Self-adaptive regulator is given the feedback according to state variation of DCLM and steady state error of motor speed, which constantly adjust the controller of pole assignment of state feedback and the device of equivalent compensation until motor output error is adjusted to become zero. Results of simulation and experiment show that higher dynamic and static performances of DCLM can be obtained by using pole assignment algorithm of self-adaptive state space design in this paper. By means of this algorithm, control quality and robustness of system have been improved.

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